Solutions

3.3: Higher-Order Linear, Homogeneous Equations with Constant Coefficients Combinations of Roots

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Exercise 1. Find the general solution to the 5th-order linear, homogeneous equation whose characteristic equation is given by

$$(r^2 + 4)^2(r^2 - 4r + 5)r^3 = 0.$$

$(r^2+4)=0 \Rightarrow r = \pm 2i$

$(r^2-4r+5)=0 \Rightarrow r = 2 \pm i$

$r^3 = 0 \Rightarrow r = 0$

Roots $= 0, 0, 0, 2 \pm i, \pm 2i, \pm 2i$

$$y = C_1 + C_2 x + C_3 x^2 + e^{2x}(a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin 2x) + x(a_3 \cos 2x + b_3 \sin 2x).$$

Exercise 2. Find the general solution to the 11th-order linear homogeneous equation whose characteristic equation has roots $3, -5, 0, 0, 0, 0, -5, 2 \pm 3i$ and $2 \pm 3i$.

$$y = (c_1 + c_2 x + c_3 x^2 + c_4 x^4) + c_5 e^{3x} + (c_6 e^{-5x} + c_7 \cdot x e^{-5x})$$
$$+ e^{2x}(a_1 \cos 3x + b_1 \sin 3x) + x e^{2x}(a_2 \cos 3x + b_2 \sin 3x).$$